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EFFECT OF VARIABLE SUCTION ON FREE CONVECTION FLOW BETWEEN TWO VERTICAL WALLS FILLED WITH POROUS MEDIUM IN SLIP FLOW REGIME

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ABSTRACT

The unsteady free convective viscous incompressible flow between two vertical walls with variable suction in slip-flow regime has been studied. Velocity and temperature profiles are obtained by using perturbation technique analytically. The expressions for skin-frictions are also derived. The effect of Darcy number, ratio of viscosities, slip parameter and Grashof number on velocity and Prandle number on temperature have shown graphically.

Keywords: *Darcy number Variable suction, vertical channel, perturbation technique, porous medium.*

I. INTRODUCTION

Free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. A series of investigations have been made by different scholars where the porous medium is either bounded by horizontal or vertical surfaces. Such phenomenon is observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth, etc. Free convective flows with periodic permeability through highly porous media play an important role in chemical engineering, turbo-machinery and in aerospace technology. Such flow include several practical applications, for example, geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport.

The recent books of Ingham and Pop [1] and Nield and Bejan [2] have documented exhaustive work on this area. The systematic studies of transport process as a result of the free convection flow between two parallel vertical walls have made by Ostrach [3], [4] and [5]. Transient free convection from a vertical flat plate has studied by Siegel [6]. Rudraih and Nagraj [7] have studied analytically the combined effect of Darcy and viscous resistances on fully developed natural convection flow of a fluid through a vertical porous stratum. Burch [8] has discussed laminar natural convection in a vertical channel. The effect of rotation encountered in nature has been analyses by Sacheti and Singh [9]. Jha [10] studied transient natural convection through vertical porous stratum.

Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction has studied by Kim [11]. Paul, Singh and Mishra [12] have examined transient natural convection between two vertical walls filled with a porous material having variable porosity. Sahin [13] studied oscillatory three dimensional flows and heat and mass transfer through a porous medium in presence of periodic suction.

Uwanta, Hanmza and Ibrahim [14] have examined heat and Mass Transfer Flow through a Porous Medium with Variable Permeability and Periodic Suction. Mixed Convection Flow Through A Porous Medium Bounded By Two Vertical Walls With Slip Boundary Conditions has examined by Mishra [15]Transient free convection flow past a vertical plate through porous medium with variation in slip flow regime has studied by Mishra, Abdullahi and Manjak [16].

Effect of Darcy number, slip parameter and ratio of viscosities on velocity and temperature profiles have been examined due to the variable suction in this paper. The governing equations are solved by using regular perturbation technique.

The behavior of velocity, temperature and skin friction for different values of physical parameters have been obtained and the results are presented graphically.

II. MATHEMATICAL ANALYSIS

An unsteady free convective flow of a viscous incompressible flow between two vertical walls through porous medium in slip-flow regime, with variable suction $V' = -V_0'(1+\varepsilon e^{\omega t})$ is considered. We introduce a co-ordinate system with wall lying vertically in $x' - y'$ plane. The x' -axis is taken in vertically upward direction along the vertical porous plate and y' axis is taken normal to the plate. The temperature of the wall at $y' = 0$ is taken T_h' whereas at the other wall $y' = H$ it is taken T_0' ($T_h' > T_0'$). Since the plate is considered infinite in the x' direction, hence all physical quantities will be independent of x'. Under these assumption, the physical variables are function of y' and t' only. Then governing equations after neglecting viscous dissipation and under usual Boussinesq's approximation can be given by the following set of equations:

$$\frac{\partial u'}{\partial t'} - V_0'(1+\varepsilon e^{\omega' t'})\frac{\partial u'}{\partial y'} = g\beta(T'-T_0') + \frac{\mu_{eff}}{\rho}\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu_f}{\rho}\frac{u'}{K'}$$
$$\rho\frac{\partial T'}{\partial t'} - V_0'(1+\varepsilon e^{\omega' t'})\frac{\partial T'}{\partial y'} = k\frac{\partial^2 u'}{\partial y'^2} \quad (1)$$

The boundary conditions are as follows

$$u' = L'\left(\frac{\partial u'}{\partial y'}\right), \quad T' = T_h' \quad at\ y' = 0$$
$$u' = 0, \quad T' = T_0' \quad at\ y' = H \quad (2)$$

In non-dimensionalisation process of the governing equations, the following dimensionless quantities are used

$$y = \frac{y'}{H}, \quad \text{t}=\frac{t'\upsilon}{H^2}, \quad \text{u}=\frac{u'H}{\upsilon},$$
$$\omega=\frac{\omega'}{\upsilon H^2}, \quad \theta=\frac{T'-T_0'}{T_h'-T_0'}$$
$$Gr = \frac{g\beta H^3(T_h'-T_0')}{\upsilon^2}, \quad \text{Rv}=\frac{\mu_{eff}}{\mu_f}, \quad (3)$$
$$\text{Da}=\frac{K}{H^2}, \quad \text{Pr}=\rho C_p / k$$

The governing equations in the non-dimensional form are as follows:

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$$\frac{\partial u}{\partial t}-(1+\varepsilon e^{\omega t})\frac{\partial u}{\partial y}=Rv\frac{\partial^2 u}{\partial y^2}+Gr\theta-\frac{u}{Da} \qquad (4)$$

$$\frac{\partial \theta}{\partial t}-(1+\varepsilon e^{\omega t})\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad (5)$$

The Boundary conditions in dimensionless form are as follows:

$$u=h\left(\frac{\partial u}{\partial y}\right), \quad \theta=1, \quad \text{at } y=0$$
$$u=0, \quad \theta=0 \quad \text{at } y=1 \qquad (6)$$

III. SOLUTION OF THE PROBLEM

Assuming the small amplitude oscillations ($\varepsilon << 1$), we can represent the velocity u and temperature θ, near the plate as: By using perturbation technique

$$u(y,t)=u_0(y)+\varepsilon e^{\omega t}u_1(y)$$
$$\theta(y,t)=\theta_0(y)+\varepsilon e^{\omega t}\theta_1(y) \qquad (7)$$

Substituting the above equation (7) in (4), (5) and (6) to get zeroth order, first order and the boundary conditions respectively

$$Rv\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}-\frac{u_0}{K}+Gr\theta_0=0 \qquad (8)$$

$$\frac{d^2\theta_0}{dy^2}+\Pr\frac{d\theta_0}{dy}=0 \qquad (9)$$

$$Rv\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial u_1}{\partial y}-(\frac{1}{Da}+\omega)u_1=-Gr\theta_1-\frac{\partial u_0}{\partial y} \qquad (10)$$

$$\frac{\partial^2 \theta_1}{\partial y^2}+\Pr\frac{\partial \theta_1}{\partial y}-\omega\Pr\theta_1=-\Pr\frac{\partial \theta_0}{\partial y} \qquad (11)$$

$$u_0=h(\frac{\partial u_0}{\partial y}),\ u_1=h(\frac{\partial u_1}{\partial y}) \quad \theta_0=1, \theta_1=0 \quad \text{at } y=0$$
$$u_0=u_1=0, \quad \theta_0=\theta_1=0 \quad \text{at } y=1 \qquad (12)$$

By solving equations (8), (9), (10) and (11) subject to boundary conditions (12), the solution for zero and first orders are obtained as follows:

$$u_0 = C_1 e^{-m_1 y} + C_2 e^{-m_2 y} + A_1 + A_2 e^{-\Pr y} \tag{13}$$

$$\theta_0 = \frac{1}{(e^{-\Pr} - 1)} \left(e^{-\Pr} - e^{-\Pr y} \right) \tag{14}$$

$$u_1 = C_5 e^{-m_5 y} + C_6 e^{-m_6 y} + A_3 e^{-m_1 y} + A_4 e^{-m_2 y} + A_5 e^{-m_3 y} + A_6 e^{-m_4 y} + A_7 e^{-\Pr y} \tag{15}$$

$$\theta_1 = C_3 e^{-m_3 y} + C_4 e^{-m_4 y} + \frac{\Pr}{\omega} e^{-\Pr y} \tag{16}$$

where

$$m_1,\ m_2 = \left(\frac{1}{Rv} \mp \sqrt{\frac{1}{Rv^2} + \frac{4}{RvDa}}\right)/2,\ m_3,\ m_4 = \left(\Pr \mp \sqrt{\Pr^2 + 4\omega}\right)/2,$$

$$m_5,\ m_6 = \left\{ \frac{1}{Rv} \mp \sqrt{\frac{1}{Rv^2} + \frac{4}{Rv}\left(\frac{1}{Da} + \omega\right)} \right\}/2$$

$$A_1 = -\frac{GrDae^{-\Pr}}{e^{-\Pr} - 1},\ A_2 = \frac{1}{(e^{-\Pr} - 1)(\Pr^2 - \frac{Rv}{\Pr} - \frac{1}{DaRv})}$$

$$A_3 = \frac{m_1 C_1}{Rvm_1^2 - m_1 - (\frac{1}{Da} + \omega)},\ A_4 = \frac{m_2 C_2}{Rvm_2^2 - m_2 - (\frac{1}{Da} + \omega)}$$

$$A_5 = -\frac{GrC_3}{Rvm_3^2 - m_3 - (\frac{1}{Da} + \omega)},\ A_6 = -\frac{GrC_4}{Rvm_4^2 - m_4 - (\frac{1}{Da} + \omega)}$$

$$A_7 = \frac{\Pr(A_2 - Gr)}{Rv\Pr^2 - \Pr - (\frac{1}{Da} + \omega)},\ A_8 = m_1 A_3 + m_2 A_4 + m_3 A_5 + m_4 A_6 + \Pr A_7$$

$$A_9 = A_3 + A_4 + A_5 + A_6 + A_7,\ A_{10} = A_3 e^{-m_1} + A_4 e^{-m_2} + A_5 e^{-m_3} + A_6 e^{-m_4} + A_7 e^{-\Pr}$$

$$C_1 = -(e^{-m_2} C_2 + A_1 + e^{-\Pr} A_2)/e^{-m_1},$$

$$C_2 = \frac{\left\{ e^{-\Pr}(1 + m_1 h) - (1 + \Pr h) e^{-m_1} \right\} A_2 + \left\{ (1 + m_1 h) - e^{-m_1} \right\} A_1}{e^{-m_1}(1 + m_2 h) - (1 + m_1 h) e^{-m_2}}$$

$$C_3 = \frac{\Pr/\,\omega(e^{-m_4} - e^{-\Pr})}{(e^{-m_3} - e^{-m_4})},\ C_4 = \frac{-\Pr/\,\omega(e^{-m_3} - e^{-\Pr})}{(e^{-m_3} - e^{-m_4})}$$

$$C_5 = \frac{-(A_{10} + C_6 e^{-m_6})}{e^{-m_5}},\ C_6 = \frac{A_{10}(1 + m_5 h) - (A_8 + A_9) e^{-m_5}}{e^{-m_5}(1 + m_6 h) - (1 + m_5 h) e^{-m_6}}$$

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The expressions for the skin-frictions at both the walls y=0 and y=1 are as follows:

$$\tau = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{du_1}{dy}\right)_{y=0} e^{\omega t}$$

$$\tau\big|_{y=0} = -\begin{Bmatrix} (C_1 m_1 + C_2 m_2 + A_2 \Pr) \\ +\varepsilon e^{\omega t}(C_5 m_5 + C_6 m_6 + m_1 A_3 + m_2 A_4 + m_3 A_5 + A_6 m_4 + \Pr A_7) \end{Bmatrix} \tag{17}$$

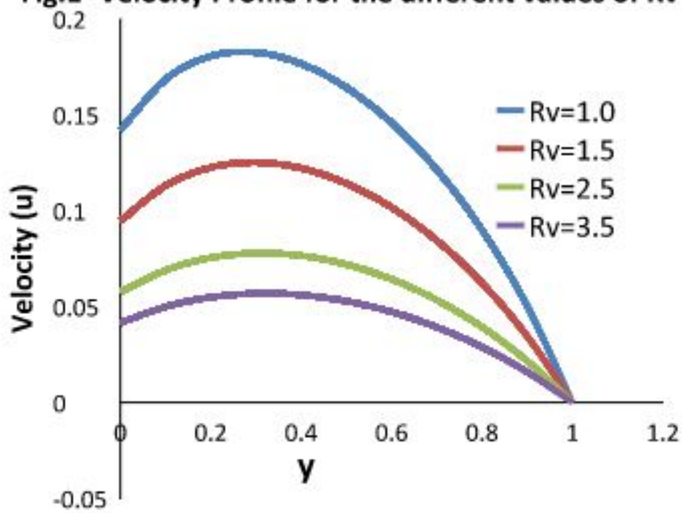
$$\tau\big|_{y=1} = -\begin{Bmatrix} (C_1 m_1 e^{-m_1} + C_2 m_2 e^{-m_2} + A_2 \Pr e^{-\Pr}) \\ +\varepsilon e^{\omega t}(C_5 m_5 e^{-m_5} + C_6 m_6 e^{-m_6} + m_1 A_3 e^{-m_1} + m_2 A_4 e^{-m_2} + m_3 A_5 e^{-m_3} + A_6 m_4 e^{-m_4} + \Pr A_7 e^{-\Pr}) \end{Bmatrix}$$

IV. RESULT AND DISCUSSION

The non-dimensional quantities emerging in the problem are the Darcy number (Da), Prandtl number (Pr), slip parameter h, Grashof number Gr and ratios of viscosities Rv. The effects of these quantities on the fluid velocity and temperature profiles have been considered and the results graphically presented and discussed in this section. In order to assess the physical inside of the problem, the effects of various parameters on Velocity distribution and Temperature distribution are studied in figures 1-5, while keeping the other parameters as constants.

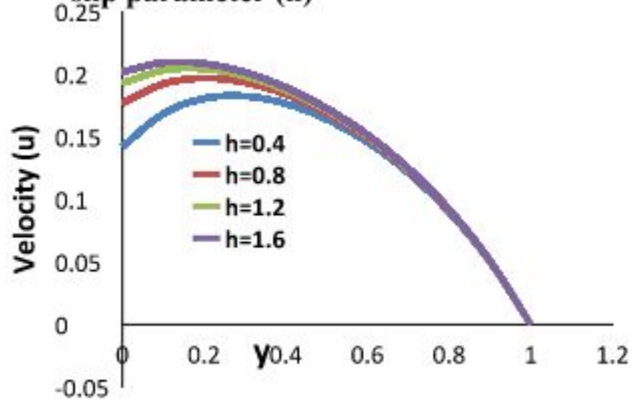
In figure 1 velocity profiles for the different values of ratio of viscosities (Rv) while all the other parameters are kept constant. It illustrate that the velocity decreases with the increase of Rv, which is due to the high effective viscosity of the porous medium to that of fluid viscosity

Fig.1- Velocity Profile for the different values of Rv



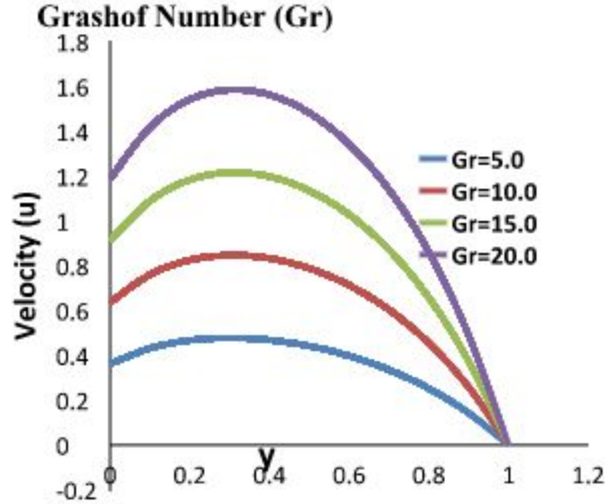
A velocity profile for different values of slip parameter (h) is shown in figure 2. It is observed that the significance of the velocity is high near the plate and there after it decreases and reaches to the stationery position at the other side of the plate. As expected velocity increases with an increase in slip parameter h.

Fig.2- Velocity Profiles for different values of slip parameter (h)



The effects of Grashof number Gr on velocity distribution are presented in figure 3. It is clear from the figure that the velocity increases with the increase of Grashof number.

Fig.3- Velocity Profile for different values of Grashof Number (Gr)



In figure 4, the effect of the variation of Darcy number (Da) is shown, which depicts that velocity increases with increase of Darcy number. It can be seen that decreasing Da decreases the porosity of the system and hence suppressed the fluid flow.

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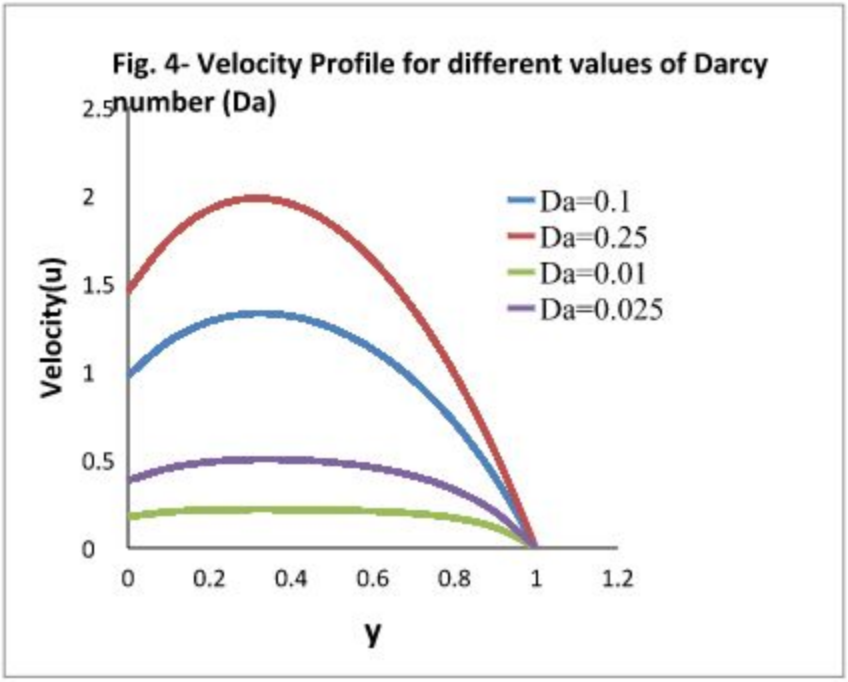
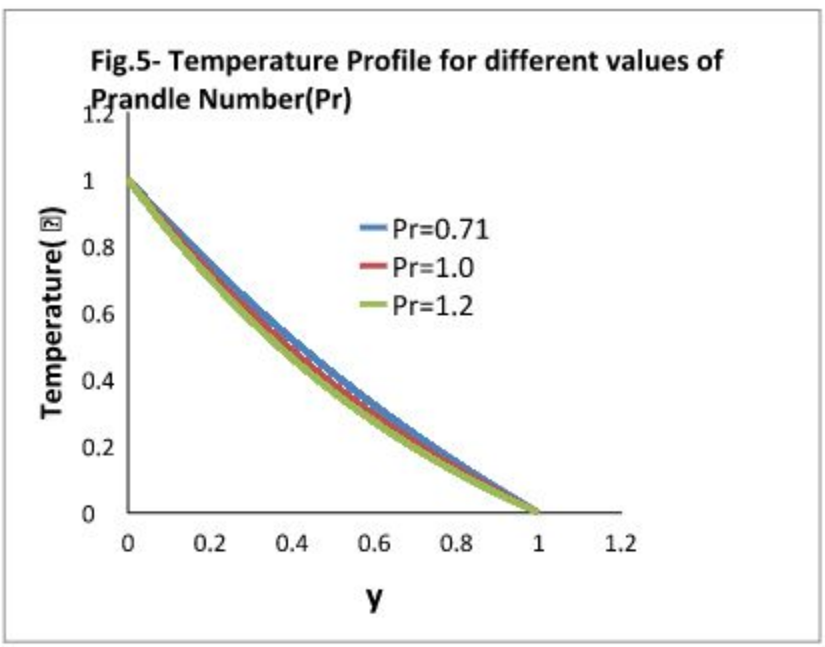


The effect of Prandle number (Pr) on temperature profile is shown in the figure 5. It indicates that temperature decreases with the increase of Pr. This behavior is as a result of the decrease in the fluid thermal conductivity as Pr increases.



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